MATH

Unpacking the Standards for Mathematical Practice

KANSAS STATE DEPARTMENT OF EDUCATION

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Unpacked Standards

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

UNPACKING THE STANDARDS FOR MATHEMATICAL PRACTICE

Pre-Kindergarten through Grade 2

1. Make sense of problems and persevere in solving them

Mathematically proficient elementary students explain to themselves and others the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway.

Example: Young students might use concrete objects or pictures to show the actions of a problem, such as by counting out and joining two sets to solve an addition problem.

2. Reason abstractly and quantitatively

Mathematically proficient elementary students make sense of quantities and their relationships in problem situations by using images or stories (contextualizing). They interpret symbols as having meaning, not just as directions to carry out a procedure.

Example: Young students might visualize the expression 11 – 8 by thinking, "If I have 8 marbles and Marie has 11, how many more do I need to have as many as Marie?" Then, in that context, they may think, "If I add 2 more that will get me to a total of 10, and then 1 more will get me to 11, so the answer is 3."

3. Construct viable arguments and critique the reasoning of others

Mathematically proficient elementary students construct verbal and written mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. Arguments may also rely on definitions, previously established results, properties, or structures. As they articulate and justify generalizations, students consider to which mathematical objects (numbers, shapes, etc.) their generalizations apply

Example: Young students might discuss with a partner that a closed shape with three sides is a triangle and why changing its color or size does not make its name change.

4. Model with mathematics

When given a problem in a contextual situation, mathematically proficient elementary students identify the mathematical elements of the situation and create or interpret a mathematical model that shows those elements and relationships among them in order to deepen understanding and/or achieve a solution.

Example: Young students asked to describe how they would share a pan of cornbread equally among 4 people might use a picture of a rectangle to represent the cornbread. The rectangle divided into 4 equal pieces is a model of the essential mathematical elements of the situation.

5. Use appropriate tools strategically

Mathematically proficient elementary students consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, etc.), drawings or diagrams (number lines, tally marks, tape diagrams, tables, graphs, etc.), models of mathematical concepts, or measuring tools.

Example: Young students may link cubes to represent two quantities and then compare the two representations side by side.

6. Attend to precision

Mathematically proficient elementary students communicate precisely to others both verbally and in writing. They start by using everyday language to express their mathematical ideas, realizing that they need to select words with clarity and specificity rather than saying, for example, "it works" without explaining what "it" means. As they encounter the ambiguity of everyday terms, they come to appreciate, understand, and use mathematical vocabulary

Examples: Young students specify units of measure; label charts, graphs, and drawings; calculate accurately and efficiently; and use clear and concise notation to record their work, such as using the >, =, and < symbols when comparing quantities.

7. Look for and make use of structure

Mathematically proficient elementary students use structures such as place value, the properties of operations, and number structures to generalize about the behavior of the operations.

Example: Young students recognize that adding 1 result in the next counting number. They are identifying the basic structure of whole numbers and can use this structure to reason about "one more" or "one less."

8. Look for and express regularity in repeated reasoning

Mathematically proficient elementary students look for regularities as they solve multiple related problems, then identify and describe these regularities.

Example: Young students may notice the pattern that occurs when finding all of the number combinations that add up to twelve (11 + 1, 10 + 2, 9 + 3, 8 + 4, etc.).

UNPACKING THE STANDARDS FOR MATHEMATICAL PRACTICE

Grades 3 - 5

1. Make sense of problems and persevere in solving them

Mathematically proficient elementary students explain to themselves and others the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. As they work, they continually ask themselves, "Does this make sense?" When they find that their solution pathway does not make sense, they look for another pathway that does. They may also consider simpler forms of the original problem.

Example: When solving a problem involving multi-digit numbers, students might first consider similar problems that involve multiples of ten or one hundred.

2. Reason abstractly and quantitatively

Mathematically proficient elementary students make sense of quantities and their relationships in problem situations. They contextualize quantities and operations by using images or stories. They interpret symbols as having meaning, not just as directions to carry out a procedure. They can then interpret the solutions to operations in terms of the context.

Example: Students might visualize the expression 40–26 by thinking, "If I have 26 marbles and Marie has 40, how many more do I need to have as many as Marie?" Then, in that context, they may think, "4 more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14."

3. Construct viable arguments and critique the reasoning of others

Mathematically proficient elementary students construct verbal and written mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. Arguments may also rely on definitions, previously established results, properties, or structures. Students consider to which mathematical objects (numbers, shapes, etc.)their generalizations apply.

Example: A student might argue that two different shapes have equal area, based on the previously demonstrated fact that both shapes are half of the same rectangle.

4. Model with mathematics

When given a problem in a contextual situation, mathematically proficient elementary students identify the mathematical elements of the situation and create or interpret a mathematical model that shows those elements and relationships among them to deepen understanding and/or achieve a solution.

Example: When students encounter situations, such as sharing a pan of cornbread among 6 people, they might first show how to divide the cornbread into 6 equal pieces using a picture of a rectangle. When the students learn to write the name of each piece in relation to the whole pan as 1 6 , they are modeling the situation with mathematical notation.

5. Use appropriate tools strategically

Mathematically proficient elementary students consider the tools that are available when solving a mathematical problem, whether in a realworld or mathematical context. These tools might include physical objects (place value manipulatives, fraction bars, etc.), drawings or diagrams (number lines, tape diagrams, graphs, etc.), models of mathematical concepts, rulers and other measuring tools, virtual manipulatives, or other available technologies

Example: Example: Students may use graph paper to find all the possible rectangles that have a given perimeter by drawing or cutting out the possible solutions.

6. Attend to precision

Mathematically proficient elementary students communicate precisely to others both verbally and in writing. They start by using everyday language to express their mathematical ideas, realizing that they need to select words with clarity and specificity. As they encounter the ambiguity of everyday terms, they come to appreciate, understand, and use mathematical vocabulary and symbols.

Examples: Students might specify units of measure; label charts, graphs, and drawings; calculate accurately and efficiently; and use clear and concise notation, such as the >, =, and < symbols, to record their work.

7. Look for and make use of structure

Mathematically proficient elementary students use structures such as place value, the properties of operations, and other generalizations about the behavior of the operations to solve problems

Example: When students calculate 16 x 9, they might apply knowledge of place value and the distributive property to find the product: 16 x 9 = (10 + 6) x 9 = (10 x 9) + (6 x 9).

8. Look for and express regularity in repeated reasoning

Mathematically proficient elementary students look for regularities as they solve multiple related problems, then identify and describe these regularities as they use algorithms to find solutions.

Example: Students might notice a pattern in the change to the product when a factor is increased by 1: 5 x 7 = 35 and 5 x 8 = 40— the product changes by 5; 9 x 4 = 36 and 10 x 4 = 40—the product changes by 4

UNPACKING THE STANDARDS FOR MATHEMATICAL PRACTICE

Grades 6 - 8

1. Make sense of problems and persevere in solving them

Mathematically proficient middle school students set out to understand a problem and then look for entry points to its solution. They analyze problem conditions and goals, translating verbal descriptions into mathematical expressions, equations, or drawings as part of the process. They consider analogous problems, and they try special cases and simpler forms of the original to gain insight into its solution.

Example: To understand why a 20% discount followed by a 20% markup does not return an item to its original price, a student might translate the situation into a tape diagram or a general equation; or they might first consider the result for an item priced at $1.00 or $10.00.

2. Reason abstractly and quantitatively

Mathematically proficient middle school students make sense of quantities and relationships in problem situations. They represent problem situations using symbols, and then manipulate those symbols in search of a solution (decontextualize). They may pause during the problemsolving process to check for meaning or to use the units of measure in the problem to clarify possible next steps (contextualize).

Example: Students may apply ratio reasoning to convert measurement units and/or proportional relationships to solve percent problems. They might then explain their answer verbally in terms of the context.

3. Construct viable arguments and critique the reasoning of others

Mathematically proficient middle school students understand and use assumptions, definitions, and previously established results in constructing verbal and written arguments. They make and explore the validity of conjectures. They can recognize and appreciate the use of counterexamples to identify common errors in algebraic manipulation.

Example: Students might debate the validity of thinking that 5 – 2x is equivalent to 3x through the use of known definitions and mathematical properties.

4. Model with mathematics

Mathematically proficient middle school students are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a given relationship and represent situations using such tools as diagrams, tables, graphs, flowcharts, and formulas. They can analyze their representations mathematically, use the results in the context of the situation, and then reflect on whether the results make sense while possibly improving the model.

Example: Students might apply proportional reasoning to plan a school event or use a set of linear inequalities to analyze a problem in the community.

5. Use appropriate tools strategically

Mathematically proficient middle school students strategically consider the available tools when solving a mathematical problem and while exploring a mathematical relationship. These tools might include pencil and paper, concrete models, a ruler, a protractor, a graphing calculator, a spreadsheet, a statistical package, or dynamic geometry software.

Example: Students might use estimation to check reasonableness, graphs to model functions, algebra tiles to see how properties of operations apply to algebraic expressions, graphing calculators to solve systems of equations, and dynamic geometry software to discover properties of parallelograms.

6. Attend to precision

Mathematically proficient middle school students communicate precisely to others both verbally and in writing. They present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound and valid reasoning, wellchosen details, and precise language.

Examples: Students might use the definition of rational numbers to explain why a number is irrational. They might also describe congruence and similarity in terms of transformations in the plane.

7. Look for and make use of structure

Mathematically proficient middle school students look closely to discern a pattern or structure. They can step back for an overview and shift perspective. This may involve viewing complicated elements as being constructed of single objects with their own contextual meanings.

Example: Students might describe their interpretation of the structure of an expression based on its individual elements—for instance, understanding 1.05a as an original value, a, plus 5% of that value, 0.05a.

8. Look for and express regularity in repeated reasoning

Mathematically proficient middle school students notice if calculations are repeated and look for both general methods and shortcuts. They continually evaluate the reasonableness of their intermediate and results.

Example: Working with tables of equivalent ratios, students might deduce the corresponding multiplicative relationships and generalize about relationships to rates.

UNPACKING THE STANDARDS FOR MATHEMATICAL PRACTICE

High School

1. Make sense of problems and persevere in solving them

Mathematically proficient high school students analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution.

Example: Students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.

2. Reason abstractly and quantitatively

Mathematically proficient high school students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize— to abstract a given situation and represent it symbolically—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Example: Students might mechanically manipulate a function to reveal zeros and other key features, then pause to consider the meaning of these features in the context of the relationship the function represents.

3. Construct viable arguments and critique the reasoning of others

Mathematically proficient high school students understand and use stated assumptions, definitions, and previously established results in constructing verbal and written arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.

Example: Students might write explanatory texts that convey their mathematical analyses and thinking, using relevant and sufficient facts, concrete details, quotations, and coherent development of ideas.

4. Model with mathematics

Mathematically proficient high school students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that future revision may be needed. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, twoway tables, graphs, flowcharts, and formulas.

Example: A student might use geometry to solve a bridge design problem or use a function to describe how the number of engines assembled depends on the number of person-hours.

5. Use appropriate tools strategically

Mathematically proficient high school students consider the available tools when solving a mathematical problem. Proficient students are sufficiently familiar with tools appropriate for high school to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from each and its limitations.

Example: Students might analyze graphs of functions and solutions generated using a graphing calculator.

6. Attend to precision

Mathematically proficient high school students communicate precisely to others both verbally and in writing, adapting their communication to specific contexts, audiences, and purposes. They develop the habit of using precise language, not only as a mechanism for effective communication but also as a tool for understanding and solving problems.

Examples: A student might describe the behavior of a function using key terms such as intercepts, increasing, maximum, and line of symmetry.

7. Look for and make use of structure

Mathematically proficient high school students look closely to discern a pattern or structure. They can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

Example: Students might describe 5 – 3(x– y) 2 as 5 minus a positive number times a square, and use that description to realize that the expression's value cannot be more than 5 for any real numbers x and y

8. Look for and express regularity in repeated reasoning

Mathematically proficient high school students notice if calculations are repeated, and they look for both general methods and shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details and continually evaluating the reasonableness of their intermediate results.

Example: Students might notice the regularity in the way terms sum to zero when expanding (x – 1)(x + 1), (x – 1)(x 2 +x + 1), and (x – 1)(x 3 + x 2 + x + 1)— leading them to the general formula for the sum of a geometric series.

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SUCCESS DEFINED

A successful Kansas high school graduate has the

- Academic preparation,
- Cognitive preparation,
- Technical skills,
- Employability skills and
- Civic engagement

to be successful in postsecondary education, in the attainment of an industry recognized certification or in the workforce, without the need for remediation.

OUTCOMES

- Social-emotional growth
- Kindergarten readiness
- Individual Plan of Study
- Civic engagement
- Academically prepared for postsecondary
- High school graduation
- Postsecondary success

Kansans CAN

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To prepare Kansas students for lifelong success through rigorous, quality academic instruction, career training and character development according to each student's gifts and talents.

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